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CASE STUDY OF INVENTORY MODEL FOR DETERIORATING ITEMS UNDER REWORK AND SHORTAGE

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Abstract

Inventory management is an important part in any industry which have inventory in the form of raw material, finished good and semifinished goods. Mini companies can save large amount of money by using inventory management techniques. It is important for every company to reduce the loss and keep the cost of finished good very competitive but if we consider deteriorating item, it is very necessary to rework on it so as to save various cost associated with it. Also continuous supply of material, smooth production, change in prices, stock out, minimising inventory cost, lot purchase are the function of inventory cost.

Various literature related to inventory control have been studied which determines in actual practice fabrication of defective items is to be anticipated due to the deterioration process. The EPQ model problem for single item under the assumption of imperfect production and perfect work have been broadly carrie out in past decades. In real manufacturing process, quality of rework is always imperfect. Various items which comes under deteriorating inventory model are electronic products, fashion clothing, pharmaceuticals, paper based material, foods, vegetable, fruits and chemical.

Case study is carried out in BEC Foods and Private Limited, Kuthrel Durg (C.G.). Objective of the research is to analyse and create an inventory model under consideration of failure rework can shortages. Also to find optimum replacement runtime and perform sensitivity analysis for the case study.

Methodology involves mathematical formulation which consists of calculation of various Time, Total cost and Inventory Size associated with inventory model under rework and shortages. At every steps while calculating the optimum cost, rework time the condition of optimum inventory also need to be calculated. Also Sensitivity analysis is carried out by changing the value of penalty cost of selling deterioted items, production setup cost, shortage production setup cost, serviceable item holding cost, defective item holding cost. Optimum solution of inventory model is shown in table in the paper which concludes that increase in penalty cost lead to increase in optimal shortage accumulation time, optimal complete backlogging time and minimum total cost. Optimal replenishment runtime, optimal rework run time, optimum non production runtime, optimum replenishment cycle time and optimum economic production quantity decreases. Similarly sensitivity analysis is done for other factor also.

In future various case study can be made under consideration of backlogging, inflation, markdown policy, variable demand, multiple production system and preservation technology.

Keywords: *EPQ, ELDSP, EMQ, MRP, EOQ, Inventory Control, Remanufacturing Goods*.

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1. Inventory Control

Inventory is coordination and supervision of the supply, storage, distribution, and recording of materials to maintain quantities adequate for current needs without excessive oversupply or loss. The inventory is used to represent the aggregate of those items of tangible assets which are:

Held for sale in ordinary course of the business.

In process of production for such sale.

To be currently consumed in the production of goods or services to be available for sale.

The inventory may be classified into three categories:

Raw material and supplies: It refers to the unfinished items which go in the production process.

Work in Progress: It refers to the semi-finished goods which are not 100% complete but some work has been done on them.

Finished goods: It refers to the goods on which 100% work has been done and which are ready for sale.

1.1 Meaning of Inventory Control

Inventory control means efficient management of capital invested in raw materials and supplies, work-in-progress and finished goods. Inventory control is the activity concerned with the management of inventory situations. The control and maintenance of inventories of physical goods is a problem common to all enterprises in any sector of a given economy. For example, inventories must be maintained in agriculture, industry, retail establishments, etc. The fundamental reason for the organizations to maintain inventories of goods is that it is either physically impossible or economically unsound to have goods arrive in a given system precisely when demands for them occur. Without inventories, customers would have to wait until their orders were filled from a source or were manufactured. In general, customers cannot wait for long periods of time. For this reason, carrying and controlling of inventories is necessary to almost all organizations that supply physical goods to customers.

The need for stocking any item arises because of the imbalance between the supply and demand of that commodity at a particular instant of time. In a typical industry, inventories of the following items become necessary:

1. Raw materials
2. Spare parts (or) repaired parts
3. Intermediate goods which are

a) Manufactured in house

b) Purchased from outside

4. Finished goods

2. Literature Review

Inventory control is the foundation of stock administration and the real fret of all undertakings in any part of a given economy. It has increased massive progression in the present economy and is common among producers, industrialists, agriculturists, specialist organizations, etc. A few researchers have committed their work on developing inventory models to get rules for working inventory framework rapidly. As far back as Harris [35] established the framework for inventory administration by developing an economic order quantity model, researchers have reinforced the literature on inventory models by calibrating certain presumptions to draw nearer to certifiable cases. A meticulous mathematical analysis of a straightforward kind of inventory model was tended by Arrow et al. [1]. From that point forward a lot of research has been completed in breaking down inventory frameworks by creating mathematical models to assist the decision makers associated with supply chains. Over a hundred years, heaps of research articles and books on inventory hypothesis and the management have been developed concentrating on an assortment of practical business circumstances. Full length book dedicated to the mathematical properties of inventory systems is that of Arrow et al. [2]. In 1963, Hadley and Whitin [34] dedicated a full length book on “Analysis of Inventory Systems”. In 1996, Naddor [73] authored a book on “Inventory Systems”. Along these lines, the works in the area of inventory emerged and gradually developed. A lot of papers and books on inventory theory and management have been published to present numerous models over the past decades that describe a variety of conditions and assumptions. This section shows a concise audit of the related writing on the connection between this theory and the general scientific classification of inventory control look into.

2.1. Inventory model with deteriorating items

A major issue in any business transaction is that control and maintain the inventories of deteriorating items. Goods are deteriorating owing to their values go down with time. The few common examples for deteriorating items are electronic products, fashion clothing, pharmaceuticals, paper-based materials, foods, vegetables, fruits and chemicals. Therefore, in practice, the loss due to deterioration cannot be ignored. The deteriorated items cannot be repaired or replaced. Deterioration occurs due to evaporation, damage, spoilage, dryness, etc., and it reduces the quality and/or quantity of stored items. Numerous studies have been carried out to address the problems of EPQ model for deteriorating items. The inventory problem of deteriorating items was first studied by Whitin [137], in which he proposed the fashion items deteriorating at the end of the storage period. After that Ghare and Schrader [29] were the first to incorporate the idea of deterioration in inventory models. They studied an exponentially decaying inventory model with constant demand. Perumal and Arivarignan [83] presented an inventory model with two rates of production, shortages, and deterioration. Later, Bhowmick and Samanta [7] developed a deteriorating inventory model with two rates of production, shortages, and variable production cycle. Lin et al. [66] provides a mechanism for measuring the influence of two stage deterioration on the supplier’s capacity utilization for the joint economic lot sizing problem model. Mashud et al. [67] discussed a non-instantaneous inventory model for two constant deterioration rates under partially backlogged shortages. Vandana et al. [131] discussed an inventory model for non-instantaneous deteriorating items considering quadratic demand rate and shortages under trade credit policy. Inderfuth et al. [46] established an EPQ model with rework process of defective items and deteriorating recoverable items. Pour and Ghobadi [84] presented a two echelon supply chain model for deteriorating items wherein the optimal selling price, production lot size, total cycle time, number of deliveries and delivery lot size are obtained simultaneously. Teng and Chang [124] presented an EPQ model for deteriorating items considering the demand rate depends on the selling price of the products and the stock level. Tiwari et al. [127] studied a manufacturer–retailer gaming problem for deteriorating products when the retailer has limited storage capacity. Tiwari et al. [128] developed a supply chain model for deteriorating items under two-level partial trade credit with price-dependent demand and partial backordering. Shukla et al. [103] presented an economic production quantity model with defective products for deteriorating products. Benkherouf et al. [6] offered an EPQ model for deteriorating, defective items and time-varying demand over planning horizon. Cárdenas-Barrón et al. [12] studied a vendor–buyer system with multiple deliveries and rework to derive the optimal delivery policy and replenishment batch size. In their study, the proposed problem is solved under two different scenarios; the first scenario dealt with the case that only number of shipments must take discrete values and in the second scenario both number of shipments and batch sizes are discrete values. Nobil and Taleizadeh [75] presented a single machine and multi-product inventory model for defective items in which the defective products produced are reworked or they are put on auction as they are. Garg et al. [28] considered a price discounting model for non-instantaneous deteriorating items.

Many researchers assumed that the rate of deterioration as a function of time or a variable and optimized the total cost of the inventory model. Chowdhury et al. [19] presented an optimal inventory replenishment policy for deteriorating items where the rate of deterioration of items is directly proportional to time. Tadikamalla [112] developed an economic order quantity model with Gamma distributed deterioration rates. An EPQ model based on the retailer's stock level was proposed by Kaliraman et al. [52] wherein it is prescribed that the rate of deterioration is Weibull distribution and the production cost consists of raw material cost, labor cost, wear and tear cost and environmental cost. Kumar et al. [62] have developed a general inventory model for deteriorating items with probabilistic deterioration rate and ramp type demand under stock dependent consumption rate. Chung and Wee [21] considered short life-cycle deteriorating items with green product design. Sarkar and Sarkar [96] developed an inventory model for inventory dependent demand and variable deterioration rate. Sarkar et al. [97] developed an inventory model under variable deterioration rates. Mishra [68] formulated a production-inventory model for deteriorating items by considering the price dependent demand while the production depends on the rate of demand.

The researchers have developed the inventory model considering the instantaneous/non-instantaneous deteriorated items in their warehouse inventory model. Sarkar and Saren [98] investigated a warehouse inventory model for variable deterioration rate. Jaggi et al. [48] investigated a replenishment policy for non-instantaneous deteriorating items in two-warehouse facilities under inflationary conditions. Chandra et al. [15] introduced the effect of deterioration on two warehouse inventory model with imperfect items. Pakkala and Achary [77] considered a two-warehouse model for deteriorating items with finite replenishment rate and shortages. Chung et al. [20] analyzed an EPQ model having two warehouses where one of them is rented and another one is owned. Hsieh et al. [41] determined the optimal lot size for a two warehouse system with deterioration and shortages using net present value. Recently, Sarkar [94] discussed inventory models with delay in payments having time varying deterioration rate and stock dependent demand. The EPQ models discussed in this section are used in this research to develop the EPQ models with deteriorating items under various realistic assumptions.

2.2. Inventory model allowing shortages

One idealistic assumption of the classical EPQ model is that the shortage is not permitted. In practice, due to production of defective items, screening of deteriorated items or other manufacturing issues, shortage is inevitable. Shortages may be backlogged in two ways: completely backlogged (Shortage units are completely fulfilled) and partially backlogged (shortage units are partially fulfilled). In present days, some customers are willing to wait until replenishment if the waiting time will be petite while others are intolerant and go somewhere else. To reflect the phenomenon, Chang and Dye [14] considered an inventory model for partial backlogging shortage where the backlogging rate is the reciprocal of a linear function of the waiting time. Recently, many researchers have focused on the partially/completely backlogging shortages. Roy et al. [88] developed an EPQ model for defective items with partially backlogging shortage. A deterministic inventory model with time-dependent demand and time varying holding cost under partially backlogging shortage was proposed by Mishra et al. [69]. Taleizadeh et al. [119] derived an EPQ inventory model for scrap items, rework, interrupted production process and backordering with the reason of minimizing the expected total inventory cost. Pal et al. [80] presented a model related to the model of Sarkar et al. [99] with a multi-production system for deteriorating items, ramp type demand and effect of inflation when there is a shortage in the stock under the finite time horizon. Taleizadeh et al. [121] developed four new sustainable economic production quantity models that consider different shortage situations. Hsu et al. [44] considered an EPQ model under an imperfect production process with shortages backordered. Wee et al. [135] developed a model for deteriorating items wherein shortage was completely backlogged. A two-warehouse inventory model for deteriorating items with price dependent demand under partial backlogging was discussed by Rastogi et al. [86]. Vandana et al. [132] examined an economic order quantity model for retailers fractional permissible delay in payment allied to order quantity through shortages.

3. Problem Identification

3.1 Research Gap

Based on the researches mentioned above, it can be concluded that a considerable amount of researches has been developed for multi-replenishment cycles with rework and complete backlogging shortage.

In our models, as stock out is an inevitable consequence of diverse uncertainties, we have developed the EPQ models with defective and deteriorating items by considering the shortages under some realistic assumptions.

The inventory research issues explored above plays an accentuation on giving sagacious devices to operational basic leadership and appropriation framework plan under different conditions. The finding of the research or literature review is that inventory control for deteriorating item and remanufacturing items under different condition need to be analysed. Concluding, inventory control model or case study for various conditions such as backlogging, inflation, markdown policy, variable demand, multiple production system, and preservation technology can be analysed. In this research we are doing the case study and analysis for deteriorating items under constraints of failure work and shortages.

3.2. Objective of Project

Following are the objectives of the project:-

a. To study about various types of inventory model considering deteriorating items.
b. To analyse the constrains under failure rework and shortages of goods.
c. To create an inventory model under failure rework and shortages of goods.
d. To determine the optimum replenishment run time for the inventory model under failure rework and shortages of goods.
e. To perform Sensitivity analysis for the inventory model under failure rework and shortages of goods.

4. Research Methodology

4.1 Assumptions

1. Production and demand rate are constants.
2. Rework and deterioration rates are constants.
3. Deterioration starts as soon as it comes into the inventory.
4. There is a replacement for the deteriorated items.
5. Shortages are permitted.
6. Demand during shortages is completely backordered.
7. Model is considered under finite time horizon.
8. The production rate of serviceable items and rework must be greater than the demand rate.
9. No machine breakdown occurs in the production run and rework run.
10. Inspection cost is negligible when compared with other costs.
11. Setup time for rework process is zero.
12. Unrecoverable defective items and deteriorated items are disposed.

4.2. Nomenclatures

4.2.1. Parameters

d	Demand rate (unit/year)
θ	Deterioration rate (unit/ year) $(0 < \theta < 1)$

α Percentage of good quality items $(0 < \alpha < 1)$

α_r Percentage of recoverable items $(0 < \alpha_r < 1)$

γ Percentage of deteriorated items screened out from the inventory

m Number of replenishment cycle.

p_c Penalty cost of selling deteriorated items to customers (Rs/unit)

k_s Production setup cost (Rs/setup)

k_p Production setup cost in a shortage (Rs/setup)

h_s Serviceable items holding cost (Rs/unit/ year)

h_r Defective items holding cost (Rs/unit/year)

4.2.2. Variables

I_1 Inventory level of serviceable items in a production run period

I_2 Inventory level of serviceable items in a rework process period

I_3 Inventory level of serviceable items in a non-production run period

I_4 Inventory level during shortage accumulating period

I_5 Inventory level during shortage backlogging period

I_{r2} Inventory level of defective items in a rework process period

$I_{r5,1}$ Inventory level of defective items in a production run period

I_s Inventory level of serviceable items at the end of a production run period

I_{ms} Maximum inventory level of serviceable items at the end of a rework process

I_{mr} Maximum inventory level of defective items in a production run period

T_1 Regular production run time

T_2 Rework run time

T_3 Regular non-production run time

T_4 Shortage accumulating time

T_5 Shortage backlogging time

T Length of the inventory cycle time

TC_F Total cost function per unit time

4.3. Formulation of the inventory model

The behaviors of the inventory level of serviceable and defective items for 3-replenishment runs are illustrated in Figure 4.2 and Figure 4.3 respectively.

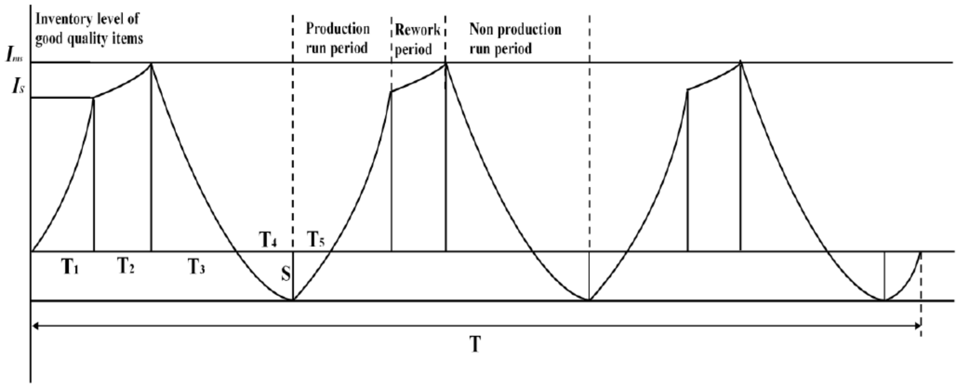

Figure 4.2. Graphical representation of the inventory behavior of good quality items with 3 replenishment cycles.

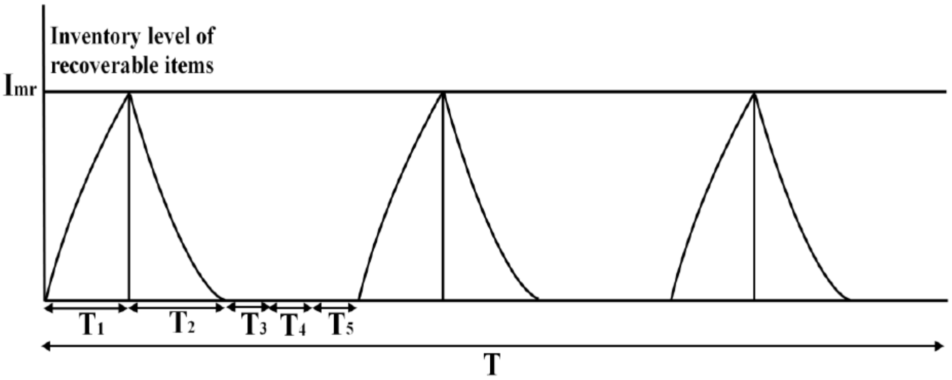

Figure 4.3. Graphical representation of recoverable items with m = 3.

The total cost function (TC_F)= deterioration cost + penalty cost of selling deteriorated items to customers + holding costs of serviceable and defective items + setup cost for a cycle + production cost + rework cost + scrap cost + shortage cost.

Now, the total cost function (TC_F) (equation (4.28)) may be represented by

The total cost function (TC_F) is a function of T_1 and T.

Solution procedure

In order to obtain the optimal solution of the model, a proof of the convexity of the objective function $TC_F(T_1, T)$ is provided. An optimization technique using partial derivatives is carried out to derive the optimal solutions.

Theorem 4.1. The total cost function $TC_F(T_1, T)$ is strictly convex

Proof:

TOTAL COST FUNCTION $TC_F(T_1, T)$ with respect to T_1 and T we get the following

$$\frac{\partial TC_F}{\partial T_1} = C_2 + \frac{2C_3T_1}{T} \quad \text{and} \quad \frac{\partial TC_F}{\partial T} = C_1 - \frac{K_S}{T^2} - \frac{C_3T_4^2}{T^2}$$

Solving $\frac{\partial TC_F}{\partial T_1} = 0$ and $\frac{\partial TC_F}{\partial T} = 0$, simultaneously we get

$$(T_1^*, T^*) = \left(-C_2\sqrt{\frac{K_S}{C_3(4C_1C_3-C_2^2)}},\ 2\left(\sqrt{\frac{K_SC_3}{4C_1C_3-C_2^2}}\right)\right) \tag{4.43}$$

Since $T_1^* > 0$ *and* $T^* > 0$ and , we must have $C_2 < 0$ and . $4C_1C_3 > C_2^2$

The Hessian matrix of the total cost function is given by

$$H = \begin{pmatrix} \frac{2C_3}{T} & -\frac{2C_3T_1}{T^2} \\ -\frac{2C_3T_1}{T^2} & \frac{2K_s}{T^3} + \frac{2C_3T_1^2}{T^3} \end{pmatrix}$$

Since $D_1(H) = \frac{2C_3}{T} > 0$ *and* $D_2(H) = D(H) = \frac{4C_3\,k_s}{T^4} > 0$, H is positive definite and hence $TC_F(T_1, T)$ is strictly convex

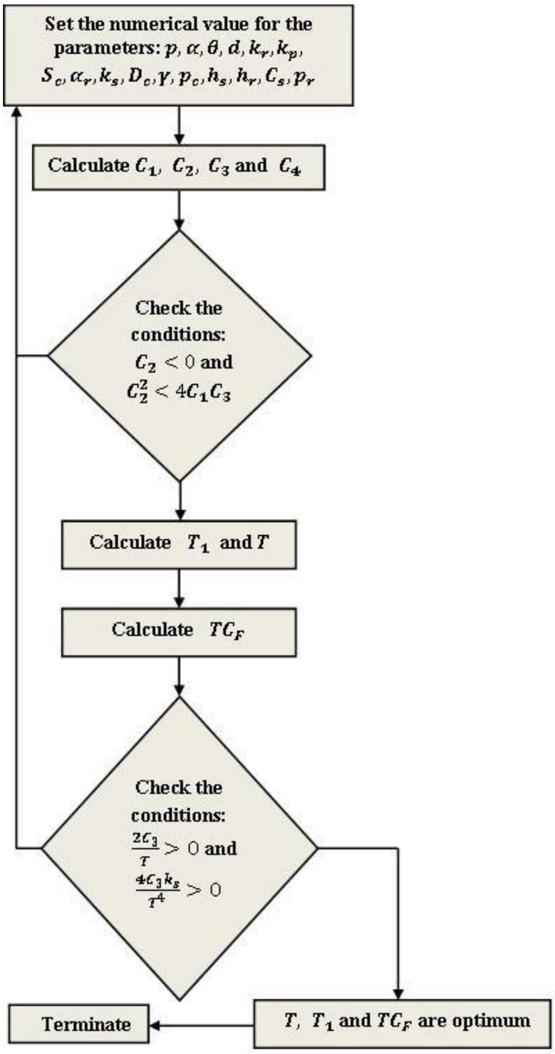

Figure 4.4: Flowchart for the solution algorithm

Algorithm 4.1

Step 1: Initialising the values of the parameters $p, \alpha, \theta, d, k_r, k_p, s_c, a_r, k_s, D_C, \gamma, p_c, h_s, h_r, C_s$ *and* p_r

Step 2: Find the total cost function $TC_F(T_1, T)$ and using step 1, find the values of constants C_1, C_2, C_3 and C_4.

Step 3: From step 2, choose one set of values of C_1, C_2 and C_3 satifying the condition: $C_2 < 0 \; and \; 4C_1C_3 > C_2^2$.
Step 4: Substitute the values of C_1, C_2 and C_3 and calculate T_1 and T.
Step 5: Substitute the values of $C_1, C_2, C_3, C_4, T_1, T$ andk_s obtained in step 2 and 4, and calculate the total inventory cost function TC_F.
Step 6: If the values of C_3 and k_s, obtained in step 2, satisfy the conditions $\frac{2C_3}{T} > 0$ and $\frac{4C_3\,k_s}{T^4} > 0$, then the corresponding values of T_1, T and TC_F obtained in step 4 and 5, are taken as the optimal production run time (denoted by T_1^*) , optimal finite planing horizon time (denoted by T^*) and minimum total inventory cost (denoted by TC_F^*) respectively. Otherwise, go to setp 2 and choose another set of values of C_1, C_2 and C_3 by changing the values of the parameters mentioned in step 1.
Step 6: Repeat the steps 2 to 6 until we get T_1^*, T^* and TC_F^*.
Step 7: End.

5. Results & Discussions

5.1 Result

Parameters are taken from the BEC Food Industry, kuthrel, Durg. Where we have received data are as follows:-
$p = 6000\,units,\; \alpha = 0.8,\; \theta = 0.1\,units, d = 2000\,units,\; k_r = 10,\; k_p = 15, s_c = 2, a_r = 0.65, k_s = 30, D_C = 3, \gamma = 0.6, p_c = 1, h_s = 3, h_r = 8, C_s = 25 \; and \; p_r = 4000\,units$

We follow from equation (4.43) that the optimal pair for two replenishment cycle is given by
$(T_1^*, T^*) = (0.0176, 0.1124)$
We obtainT_2^*, T_3^*, T_4^* and T_5^* from the equations (4.38), (4.39), (4.41) and (4.40) respectively:
$T_2^* = 0.0060, T_3^* = 0.0264, T_4^* = 0.0086$ and $T_5^* = 0.0026$
The optimal replenishment run time and the economic production quantity are represented by:
$T_p^* = T_1^* + T_5^* = 0.0202$ and $Q^* = pT_p^* \approx 121$.
The maximum inventory level of serviceable items during replenishment run time period is given by:
$I_S \approx 49$.
The maximum inventory level of serviceable items at the end of replenishment run is given by:
$I_{ms} \approx 53$.
The maximum inventory level of imperfect items at the end of replenishment run is given by:
$I_{mr} \approx 24$.
The amount of shortage per replenishment cycle is given by:
$S \approx 7$.
The corresponding optimal total inventory cost per unit time can be obtained by using equation (4.42):
$TC_F^* = Rs. 74254$

5.2 Sensitivity analysis

In this section, we study the effect of changes in the value of system parameters p_c, k_s, h_s, h_r and m on the optimal values $T_i^* (i = 1,2,3,4,5)$, optimal replenishment cycle T^*, the economic production quantity (Q^*) and the minimum total inventory cost per unit time (TC_F^*). The sensitivity analysis is performed by changing each of the parameters considered in this section taking one parameter at a time and keeping the other parameters remains unchanged. The results are given in the tables 5.1 to 5.6.

Table 5.1: Optimal solution of this inventory model for different p_c.

p_c	T_1^*	T_2^*	T_3^*	T_4^*	T_5^*	T^*	TC^*	Q^*
1	0.0176	0.0060	0.0264	0.0086	0.0026	0.1124	74254.00	121
10	0.0173	0.0060	0.0260	0.0088	0.0026	0.1113	74260.00	120
20	0.0170	0.0059	0.0256	0.0091	0.0027	0.1101	74266.00	118
30	0.0167	0.0059	0.0252	0.0093	0.0028	0.1089	74271.00	117
40	0.0165	0.0058	0.0248	0.0095	0.0028	0.1078	74277.12	116
50	0.0162	0.0057	0.0244	0.0098	0.0029	0.1067	74282.70	115
60	0.0160	0.0057	0.0241	0.0100	0.0030	0.1057	74288.16	114
70	0.0157	0.0056	0.0237	0.0102	0.0030	0.1047	74293.51s	113

Table 5.2: Optimal solution of this inventory model for different k_s.

k_s	T_1^*	T_2^*	T_3^*	T_4^*	T_5^*	T^*	TC^*	Q^*
30	0.0176	0.0060	0.0264	0.0086	0.0026	0.1124	74254.00	121
35	0.0190	0.0065	0.0285	0.0093	0.0028	0.1214	74297.00	131
40	0.0203	0.0070	0.0305	0.0100	0.0030	0.1298	74336.70	140
45	0.0215	0.0074	0.0324	0.0106	0.0031	0.1377	74374.09	148
50	0.0227	0.0078	0.0341	0.0111	0.0033	0.1451	74409.44	156
55	0.0238	0.0082	0.0358	0.0117	0.0035	0.1522	74443.07	166
60	0.0249	0.0085	0.0374	0.0122	0.0036	0.1590	74475.21	171
65	0.0259	0.0089	0.0389	0.0127	0.0038	0.1655	74506.03	178

Table 5.3: Optimal solution of this inventory model for different h_s.

h_s	T_1^*	T_2^*	T_3^*	T_4^*	T_5^*	T^*	TC^*	Q^*
3	0.0176	0.0060	0.0264	0.0086	0.0026	0.1124	74254.00	121
5	0.0139	0.0052	0.0210	0.0114	0.0034	0.0965	74342.00	104
7	0.0116	0.0047	0.0176	0.0134	0.0040	0.0870	74410.25	94
9	0.0100	0.0043	0.0153	0.0150	0.0045	0.0806	74464.92	87
11	0.0088	0.0041	0.0135	0.0163	0.0049	0.0760	74510.24	82
13	0.0078	0.0039	0.0121	0.0174	0.0052	0.0724	74548.59	78
15	0.0070	0.0037	0.0110	0.0183	0.0055	0.0697	74581.56	75
17	0.0064	0.0036	0.0100	0.0192	0.0057	0.0674	74610.26	73

Table 5.4: Optimal solution of this inventory model for different h_r.

h_r	T_1^*	T_2^*	T_3^*	T_4^*	T_5^*	T^*	TC^*	Q^*
8	0.0176	0.0060	0.0264	0.0086	0.0026	0.1124	74254.00	121
10	0.0169	0.0058	0.0254	0.0083	0.0025	0.1080	74276.21	116
12	0.0163	0.0056	0.0244	0.0080	0.0024	0.1040	74297.44	112
14	0.0157	0.0054	0.0236	0.0077	0.0023	0.1004	74317.92	108
16	0.0152	0.0052	0.0228	0.0075	0.0022	0.0972	74337.71	105
18	0.0147	0.0051	0.0222	0.0072	0.0022	0.0943	74356.90	101
20	0.0143	0.0049	0.0215	0.0070	0.0021	0.0916	74375.52	98
22	0.0139	0.0048	0.0210	0.0068	0.0020	0.0891	74393.62	96

Table 5.5: Optimal solution of this inventory model for different replenishment, m.

m	T_1^*	T_2^*	T_3^*	T_4^*	T_5^*	T^*	TC^*	Q^*
2	0.0176	0.0060	0.0264	0.0086	0.0026	0.1124	74254.00	121
3	0.0117	0.0040	0.0176	0.0057	0.0017	0.1221	111114.35	81
4	0.0088	0.0030	0.0132	0.0043	0.0013	0.1224	147974.57	60
5	0.0070	0.0024	0.0106	0.0034	0.0010	0.1220	184834.78	48
6	0.0059	0.0020	0.0088	0.0029	0.0009	0.1230	221695.00	40
7	0.0050	0.0017	0.0076	0.0025	0.0007	0.1225	258555.21	35
8	0.0044	0.0015	0.0066	0.0022	0.0006	0.1224	295415.43	30
9	0.0039	0.0013	0.0059	0.0019	0.0006	0.1224	332275.64	27

5.3 Discussion

Based on our numerical results, the following managerial phenomena can be drawn from Table 5.1 to 5.5:

1. Increase in the penalty cost (p_c) leads to increase in the optimal shortage accumulated time T_4^*, the optimal complete backlogging time T_5^* and the minimum total cost (TC_F^*). On the other hand, the optimal replenishment run time T_1^*, optimal rework-run time T_2^*, the optimal non-production run time (T_3^*), the optimal replenishment cycle time (T^*) and the optimal economic production quantity $Q^*$are decreasing. Both TC_F^* and Q^* are very lightly sensitive to the changes in the values of the parameter p_c.

2. Increase in the production setup cost (K_s) leads to increase in the optimal replenishment run time (T_1^*), optimal rework-run time (T_2^*), the optimal non-production run time (T_3^*), the optimal shortage accumulated time (T_4^*), the optimal complete backlogging time (T_5^*), the optimal replenishment cycle time (T^*), the minimum total cost (TC_F^*) and the optimal economic production quantity (Q^*). Hence, the total cost increases if the production setup cost increases. TC_F^* is slightly sensitive to the changes in the values of the parameter K_s whereas Q^* is moderately sensitive to the changes in the values of the parameter K_s.

3. If the holding cost (h_s) of serviceable items increases, then the optimal shortage accumulated time (T_4^*), the optimal complete backlogging time (T_5^*) and the minimum total inventory cost (TC^*), also increases. But the optimal replenishment run time (T_1^*), the optimal rework-run time (T_2^*), the optimal non-production run time T_3^*,

the optimal replenishment cycle time (T^*)and the optimal economic production quantity (Q^*) decreases. TC_F^* is moderately sensitive to the changes in the values of the parameter h_s while Q^* is highly sensitive to the changes in the values of the parameter h_s. That is, minimum holding cost will minimize the total inventory cost and production quantity of the manufacturer.

4. Increase in the holding cost (h_r) of imperfect items leads to decrease in the optimal replenishment run time (T_1^*), optimal rework-run time (T_2^*), the optimal non-production run time (T_3^*), the optimal shortage accumulated time (T_4^*), the optimal complete backlogging time (T_5^*), the optimal replenishment cycle time (T^*), and the optimal economic production quantity (Q^*). whereas the minimum total inventory cost (TC_F^*) increases. That is, increasing of holding cost of imperfect items reduces all the time length, therefore, economic production quantity is reduced. TC_F^* is very lightly sensitive to the changes in the values of the parameter $h_r$whereas Q^* is lightly sensitive to the changes in the values of the parameter h_r.

5. Increase in the replenishment run time leads to decrease in the optimal replenishment run time (T_1^*), optimal rework-run time (T_2^*), the optimal non-production run time (T_3^*), the optimal shortage accumulated time (T_4^*), the optimal complete backlogging time (T_5^*), the optimal replenishment cycle time (T^*) and the optimal economic production quantity (Q^*) whereas the minimum total inventory cost (TC_F^*) increases. That is, increasing of replenishment run time reduces all the time length, therefore, economic production quantity (Q^*) per replenishment cycle is reduced and the total inventory cost (TC_F^*) is increased. Both $TC^*$and Q^* are very highly sensitive with the replenishment run time.

6. It is clearly observed that if replenishment run time decreases, then economic production quantity (Q^*) per replenishment cycle also decreases and if replenishment run time increases, then economic production quantity (Q^*) per replenishment cycle also increases. Therefore, to fabricate more quantity, the manufacturer should increase replenishment run time and to fabricate less quantity, the manufacturer should reduce replenishment run time. Since holding and setup costs are moderately sensitive with total inventory cost, the manufacturer should maintain the inventory with less holding and setup costs.

This model is presented for a number of practical circumstances such as reworks, shortages and screening etc. The model is an addition to this line of research. The approach implemented here provides the practitioners with more functional alternative because it prevents the defective products through screening process and gives more accurately the economic production lot size and replenishment run time. It should be emphasized that the inspection part in the model is suitable for practitioners who have an automated screening system to prevent defective items thereby passes high quality product to the market place and the consumers.

The following figures (Figure 5.1 to Figure 5.2) show that the effectiveness of different parameters on total cost.

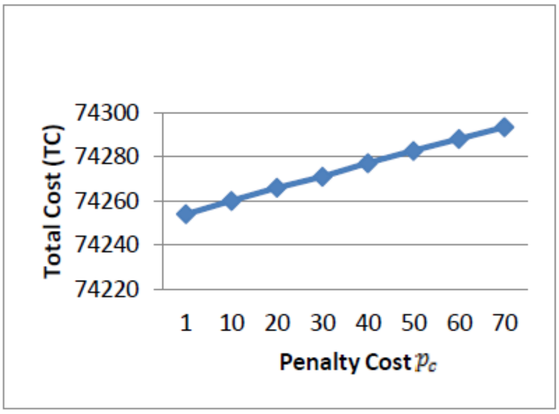

Figure 5.1 Total cost v/s p_c.

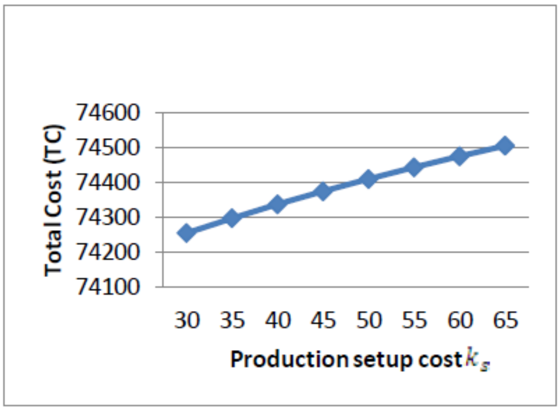

Figure 5.2. Total cost v/s k_s

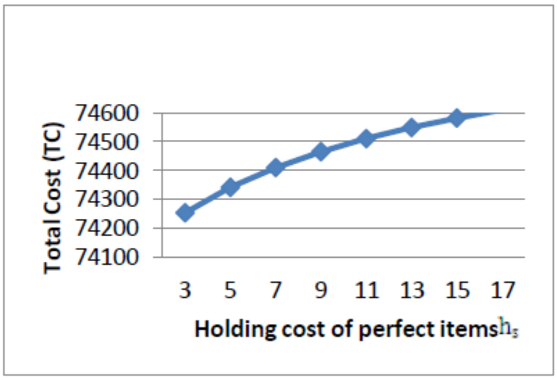

Figure 5.3. Total cost v/s h_s

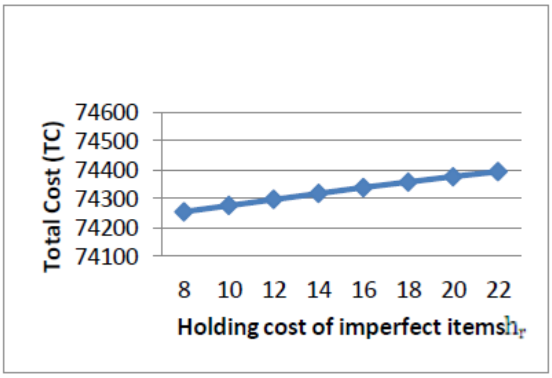

Figure 5.4. Total cost v/s h_r

The results might be utilized by managers as a reference when consider imperfect economic production lot quantity models under reworking and shortages. The manufacturer ought to sell the items as presently as attainable to avoid higher holding price. This model is given for variety of sensible problems centered by the production-oriented organizations like deterioration, defective, rework, shortage, and screening. This model is an addition to the current line of analysis. The approach enforced here provides the company managers with more functional alternative because it prevents the defective merchandise through the screening method and provides more accurately the economic production lot quantity, production run time and number of units of defective and scrap items. This model emphasized that the inspection part in the model is suitable for industry managers who have an automatic screening method to forestall the imperfect and deteriorating products. This model helps the corporate managers to supply 100 percent high-quality merchandise by eliminating defective products through screening method so high-quality products can be sold-out in marketplaces and customers, which will make a positive impact on company image.

6. Conclusion and Future Work

6.1 Conclusion

In this section, the research findings will be briefly shown in order to support the aim of this thesis in accordance with the research objectives.

In modem competitive world, the serious issue and principal responsibility of a company/industry, whether it is an public sector, private sector or government department (business or industry) is to optimize the utilize of assets. For the survival and growth of an industrial enterprise, it is highly essential that all the pervasive efforts are made to minimize and control the total costs, to attain higher operational efficiency and profitability of a company/industry. Inventory is a very important resource of a business enterprise. Inventory management is a vital technical tool for controlling inventory and eliminating wastage and it is viewed as an indispensable piece of Industrial administration in present day times.

This research has developed a modified EPQ inventory model for deteriorating items wherein multi-production setup, remanufacturing, deterioration of items with preservation technology and shortage have been considered. It has found that the economic replenishment lot size, optimal replenishment run time, optimal backordered quantity and optimal cycle length so as to minimize the total cost of an imperfect production inventory system. The products are classified into two groups which are serviceable and defective items and then the defective items are further classified into two groups which are defective but recoverable and defective but non-recoverable items. The unrecoverable items are disposed. The research has found that the number of products recoverable and that of unrecoverable with the help of the remanufacturing process.

Also, in this manufacturing/remanufacturing system, the production process contains multi-production setups and in each production setup, rework production setup is developed. Production setup cost consists of raw material cost, labor cost and environmental pollution cost. Since the production process is imperfect, the defective items are produced. The defective items are reworked as 100% original quality items and the items are sent to customers. This EPQ inventory model investigates optimal replenishment quantity, optimal number of replenishment, optimal production run time and optimal finite planning horizon time. This research also found that the production cost, setup cost and holding cost of serviceable and defective items are much affect the total inventory cost. This research presents an inventory model of direct application to the venture that consider the fact that the stored items are deteriorated during storage epoch and defective items are produced during production era.

This research dealt with an EPQ model for deteriorating items with a price dependent demand under markdown policy where the demand increases when price decreases. In practice, a retailer in a supermarket has to deal with the problem of highly perishable seasonal product where the effect of deterioration is considerable. In this study, the preservation technology is taken into account to reduce the deterioration rate during the deterioration period of instantaneous deteriorating items. This model is appropriate for those perishable items, whose demand in the market is low as compared to other items and they deteriorate continuously with time. The results exhibit that markdown offering time and markdown rate give a significant contribution to optimize the total cost per unit time and a policy maker must be exceptionally watchful to set markdown offering time and markdown rate, because optimum policy is diverse for various cases.

In this research, an imperfect EPQ model has been developed for defective items, in which four different levels of production are considered. The planned model is appropriate for freshly launched product with constant pattern up to a degree in time. Such state of affairs has fascinated since from beginning at low rate of production, large quantum of stock of manufacturing items at the initial stage can be avoided which will be led to a reduction in the holding cost. The proposed inventory model can assist the manufacturer and retailer in determining the optimal replenishment lot size, replenishment-run time, replenishment cycle length, number of replenishment cycles, number of defective items, number of scrap items, and total inventory cost accurately in a (m:4-1)production system. This model also helps the manufacturers to find the number of units of defective items produced per cycle, in which the quantity of recoverable and scrap items can be found.

Also the result of this research shows that, in a single vendor and single buyer imperfect production system, the manufacturer must deliver the finished products in small quantities to minimize the buyer's holding cost and the equal shipment interval policy is always superior to the equal shipment lot size policy.

6.1 Future Work

The work presented in this thesis has some limitations which should be investigated and overcome.

For future work, the proposed inventory models can be extended under different realistic situations viz., variable deterioration rate, probabilistic demand, power demand and fuzzy demand rate, etc. Furthermore, imperfect inspection of ensuring quality (i.e. an inspector may classify a serviceable items to be defective (Type I error) and he may also classify an imperfect item to be serviceable (Type II error)) would be taken into account.

This approach can also be extended to other problem by considering delay in payment, unit cash discount, stock-dependent demand, multi item inventory models, reliability of the items, error in inspection process and single-vendor multi-buyer. Future research could extend the current models to economic production quantity models with power demand pattern, deterioration and backlogged shortages in which the replenishment rate is not necessarily infinite. To close the gap between practice and academia, it is desired to extend this model further by considering the effect of salvage to the second market on the demand of the first market, by incorporating both selling salvage pricing decisions that influence customer demand, and by including complete backlogging or a mixture of backorders and lost sales.

The economic production quantity models could consider the effect of inflation rates on the optimal credit period and cycle time simultaneously. In this work, the inspection cost and the lead time are considered as negligible. In near future, this model could be extended by considering the inspection cost for the defective items and non-zero lead time. An extension to the model can also be done by considering the effect of the machine breakdown and the probabilistic percentage of the imperfect quality items. It would be interesting to model the problem when various parameters are not deterministic and described in fussy or interval form. Following are the future scope for the study:

a. A deterministic inventory model for deteriorating items and determining optimal replenishment run time with remanufacturing policy and complete backlogging.
b. Determining an optimal replenishment run time and lot size for an imperfect manufacturing system and rework under markdown policy.
c. A production inventory model for single vendor single buyer integrated demand with multiple production setups and remanufacturing system.

d. A manufacturing inventory model for exponentially increasing demand with preservation technology and complete backlogging.

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